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Fracture Mechanics Concepts in Reliability Analysis of Monolithic Ceramics

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FRACTURE MECHANICS CONCEPTS IN RELIABILITY ANALYSIS OF MONOLITHIC CERAMICS

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ABSTRACT

Basic design concepts for high-performance, monolithic ceramic structural components are addressed. The design of brittle ceramics differs from that of ductile metals because of the inability of ceramic materials to redistribute high local stresses caused by inherent flaws. Random flaw size and orientation requires that a probabilistic analysis be performed in order to determine component reliability. The current trend in probabilistic analysis is to combine linear elastic fracture mechanics concepts with the two parameter Weibull distribution function to predict component reliability under multiaxial stress states. Nondestructive evaluation supports this analytical effort by supplying data during verification testing. It can also help to determine statistical parameters which describe the material strength variation, in particular the material threshold strength (the third Weibull parameter), which in the past has often been taken as zero for simplicity.

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INTRODUCTION

Ceramic materials are currently under consideration for structural components in advanced heat engines for several reasons. The primary motivation is their high strength under significantly increased operating temperatures. The use of these high temperature materials will permit better fuel efficiency and reduce cooling system requirements. In addition to high temperature strength, the low density of ceramics also makes them attractive, especially for rotating components. Other enticing physical properties include good oxidation and corrosion resistance, low friction and wear characteristics, and the fact that ceramics are manufactured from abundant, nonstrategic raw materials.

Unfortunately, several undesirable characteristics are also present in ceramics because of their lack of ductility and their random flaw distribution. Due to their atomic structure, ceramics intrinsically exhibit low strain tolerance and low fracture toughness. Low toughness, when combined with desired tensile stress states, usually results in a very small critical flaw size. Furthermore, a large dispersion in fracture strength exists.

The fracture strength varies not only in nominally identical tests, but the mean strength is also dependent upon the method of testing and the volume of tested material. Limited design experience exists for materials having such properties, with much of the early brittle materials design experience gained from work with graphite (Refs. 1 and 2) or glass (Ref. 3).

There is a well-established link between linear elastic fracture mechanics (LEFM) and nondestructive evaluation (NDE) in the flaw-tolerant design method for metallic components. However, several factors currently prevent a similar situation for ceramics. First, a vast number of flaws typically exist in a varying multiaxial stress field within a ceramic component. Second, the probabilistic nature of advanced NDE methods must be taken into account, especially for the small critical flaw sizes encountered in such applications. Third, the ability of the NDE method to determine the flaw type, shape, and orientation must be considered. Fourth, the relationship between the flaw and the fracture strength must be established. There are many types of possible strength-controlling flaws: machining damage, cracks, pores, agglomerates, inclusions, large grains, grain boundaries, and other microstructural irregularities. According to a recent review by Singh (Ref. 4) on the effect of flaws on the fracture strength of ceramics, the flaw size-strength relationship is not yet determined for all flaw types in all materials. However, experimental data shows that most defects exhibit crack-like behavior and can be modeled by LEFM even though they are not true cracks in every sense, i.e., they are not flat planes of geometric discontinuity. This is done by introducing an effective crack which is related, although not exactly equal, to the actual flaw shape and size. Due to the above considerations, an a priori analytical determination of the "critical flaw" in a component is presently intractable. Hence, it is generally impractical to use traditional deterministic fracture mechanics and single crack NDE to predict the strength of a given ceramic component.

Therefore, the current trend in monolithic ceramics structural analysis is to couple LEFM with statistical weakest link concepts to predict the strength variation and failure behavior of ceramic components. In this method, a fracture strength distribution is first determined from simple uniaxially loaded (flexural or tensile) test specimens. Then, using this data, the reliability for a given component geometry and loading can be calculated. A direct correlation between flaw type/size and fracture strength is not necessary. Accordingly, pre-existent flaws do not need to be explicitly detected.

However, this does not imply that NDE cannot play a role in the reliability analysis. The objective of this paper is to discuss two of NDE's roles in the analytical prediction of fast-fracture reliability of structural ceramics, along with a basic description of the reliability analysis itself. Accurate experimental verification of the reliability models demands NDE characterization of the test specimens. A second role would be to correlate NDE measurements with the statistical parameters required for reliability analysis, thus eliminating costly testing. Furthermore, in the low failure probability regimes of interest in design, NDE provides increased analysis capability by experimentally determining a third statistical parameter to be

used in the analysis. The first role, experimental verification, is currently in progress at NASA Lewis Research Center and will be briefly summarized in a later section of this paper. The latter role, parameter determination, is problematical at this time, and is consequently seldom used, but its potential application will be described. Information about reliability analysis will be presented to aid in the understanding of NDE's roles in this research area.

RELIABILITY ANALYSIS

Fundamentals of the Weakest Link Fracture Theory

The fracture strength of ceramics is controlled by one of the larger flaws in the material and hence can be described by a form of extreme value statistics. Most mathematical descriptions of failure probability are based on the weakest link theory (WLT). The WLT assumes that complete failure will occur when the weakest link of a number of independent and mutually exclusive links fails. Consequently, if two apparently identical materials of different volumes are subjected to the same stress state, the larger volume of material will be more likely to fail because it contains more links, some of which are likely to be weaker than those found in the smaller specimen. WLT is obviously a more conservative failure theory than the parallel or bundle model which allows redistribution of loads to the surrounding material when one link fails. Because of the brittle nature of monolithic ceramics, it has been observed that they behave in a weakest link or series manner. For comparison, it is believed that composite ceramics could be described by some combination of parallel and series concepts, meaning failure of one element of material will not necessarily cause catastrophic fracture.

The weakest link concept was first proposed by Midgley and Pierce (Ref. 5) who noted that as the length of a piece of yarn increased, the strength distribution became negatively skewed, i.e., there was a greater tendency to fracture at a lower strength. Weibull applied the same concept to the strength of a solid volume of material, but in addition he assumed a unique distribution function which is now known as the Weibull distribution.

A general form of the Weibull distribution for volumetric flaws is

$$P_f = 1 - \exp\left[-\int_V \left(\frac{\sigma - \sigma_u}{\sigma_o}\right)^m dV\right] \quad (\sigma > \sigma_u) \qquad (1)$$

$$= 0 \qquad (\sigma < \sigma_u)$$

where the probability of failure, P_f, due to a uniaxial stress, σ, is dependent upon three statistical material parameters--the Weibull modulus (shape parameter) m, the threshold strength (location parameter) σ_u, and the normalizing stress (scale parameter) σ_o. The Weibull modulus is indicative of strength variability, with smaller values representing a larger variation. Ceramics typically have a modulus between 5 and 15 whereas the

shape parameter of most ductile metals is greater than 40. Generally, the strength dispersion in metals is small enough that the mean strength value is sufficient for most design purposes and a probabilistic analysis is not necessary. The normalizing stress is related to the mean strength. Note that for dimensionless P_f, the units of σ_o are stress x (volume)$^{1/m}$, not stress. The threshold strength is the maximum allowable stress for which there is no possibility of failure. By assuming that the threshold strength is zero, the two parameter Weibull model is obtained. Since this form is adequate to describe most strength data and it is mathematically simple, the two parameter form of the Weibull equation is often used for structural ceramics. Consequences of this assumption will be discussed in a later section.

The Weibull equation shows an important difference between design based on probabilistic strength distribution and a deterministic design; in the former approach the stress distribution over the entire volume of the component is necessary. The design is not necessarily governed by the the most highly stressed location or the critical point, but by the entire stress field.

A similar form of the Weibull distribution could be written for surface flaw induced fracture, with corresponding surface material parameters. For simplicity, this paper will deal only with the equations for the volume flaw population. Although not discussed herein, insight into the interaction between multiple flaw populations may be obtained from Johnson (Ref. 6).

Multiaxial Reliability Predictions

The Weibull function is usually used to characterize the strength distribution in a uniaxial stress state. It may also be used for a multiaxial stress field, provided that the normalizing stress is properly adjusted. However, this approach requires testing in each stress state for which reliability data is desired. Due to the cost involved in fabricating and testing complicated components, it is obvious that an alternate method is necessary. The objective is then to develop analysis methods so that the reliability in any stress state can be calculated using only uniaxial test data. Several approaches to the problem have been proposed.

The first of these suggestions was Weibull's arbitrary assumption that the uniaxial theory could be extended by obtaining an effective stress through averaging the tensile normal stress at a given location before performing the volume integration. This method involves the integration, over a spherical surface area with unit radius, of the stress (raised to the power m) normal to a tangent plane of the unit sphere. Thus, the normal stress in every possible direction at the point is included. Except for simple loading conditions such as uniaxial stress states, numerical integration is necessary. Under a general loading condition, compressive normal stresses are excluded from the averaging process. This method ignores the effect of shear stresses, or, in LEFM terminology, only mode I fracture is presumed to contribute to the failure probability. In spite of its limitations and its numerical requirements, the normal stress averaging method has been used extensively (Refs. 7 to 14).

A simpler method called the principle of independent action (PIA) was proposed to avoid the integration required for the averaging process. This model assumes that each tensile principal stress (σ_1, σ_2, and σ_3) acts independently of the others to give a total failure probability of

$$P_f = 1 - \exp\left\{-\int_V \left[\left(\frac{\sigma_1}{\sigma_o}\right)^m + \left(\frac{\sigma_2}{\sigma_o}\right)^m + \left(\frac{\sigma_3}{\sigma_o}\right)^m\right] dV\right\} \qquad (2)$$

in the case where $\sigma_i > 0$ $(i=1,2,3)$. This model is the statistical equivalent of the maximum principal stress theory. It has often been used for structural ceramic design (Refs. 15 to 22). However, it yields unconservative results because it ignores both the shear stresses and the interaction between principal stresses.

Recently, several theories have been proposed which model the behavior of brittle materials in a mechanistic rather than phenomenological manner. These theories explicitly assume that cracks are present in the material and assume linear elastic fracture mechanics and weakest link type behavior. The model that will be discussed in detail here was initially proposed by Batdorf and Crose (Ref. 1) and was later extended to account for shear stresses by Batdorf and Heinisch (Ref. 23).

Before discussing Batdorf's assumptions, the concepts of critical stress and effective stress, σ_{cr} and σ_e respectively, will be developed. A critical stress is defined as the remote stress which will cause fracture when applied normal to a given crack. Hence, from LEFM, the critical stress is dependent upon the crack length and shape, the material mode I fracture toughness, and the geometrical constraints surrounding the crack. Cracks are generally small enough and far enough apart that the geometry can be assumed to be a single crack in an infinite body. The effective stress acting on a crack is dependent upon the selection of a mixed-mode fracture criterion. The theories which predict initiation of crack propagation under combined normal and shear loads can be categorized as coplanar and out-of-plane crack extension theories. Coplanar theories include those that are based on critical values of normal stress, maximum tensile stress, and total strain energy release rate. Out-of-plane crack extension criteria are believed to more accurately reflect reality and they include the maximum strain energy release rate, the minimum strain energy density, and the maximum tangential stress. However, these theories lead to more complex equations for effective stress and, therefore, are often replaced by simple approximations (Ref. 24).

The derivation of a simple effective stress equation will be shown to illustrate the basic principles. Assume that mixed-mode fracture takes place when the crack extension force or strain energy release rate, G, reaches a critical value, G_c. The total strain energy release rate is related to the stress intensity factors, K_I, K_{II}, and K_{III} for opening, sliding, and tearing modes respectively by (Ref. 25)

$$G = \frac{1 - \upsilon^2}{E}\left(K_I^2 + K_{II}^2 + \frac{1}{1 - \upsilon} K_{III}^2\right) \tag{3}$$

under plane strain conditions, where E is Young's modulus and υ is Poisson's ratio. The stress intensity factors for a Griffith crack of length $2a$ in an infinite plate subjected to a normal tensile stress σ_n and a shear stress τ parallel to the length of the crack are (Ref. 25)

$$K_I = \sigma_n \sqrt{\pi a}, \quad K_{II} = \tau \sqrt{\pi a} \tag{4}$$

where σ_n and τ are determined from the principal stresses by stress transformation equations.

The effective stress is obtained by equating the value of G for mode I fracture (G_{Ic}) to its equivalent value at fracture under mixed in-plane loading conditions, that is

$$\frac{1 - \upsilon^2}{E} \sigma_{cr}^2 \pi a = \frac{1 - \upsilon^2}{E} (\sigma_n^2 \pi a + \tau^2 \pi a)$$

or

$$\sigma_{cr}^2 = \sigma_n^2 + \tau^2 \tag{5}$$

and, noting that the critical stress is the effective stress in a mode I test, we obtain

$$\sigma_e = \sqrt{\sigma_n^2 + \tau^2} \quad \text{for} \quad \sigma_n \geq 0 \tag{6a}$$

If the shear is instead assumed to be in the direction of mode III cracking, possible only for volume flaws, then using Eq. (3) and $K_{III} = \tau\sqrt{\pi a}$ we obtain

$$\sigma_e = \sqrt{\sigma_n^2 + \frac{1}{1 - \upsilon} \tau^2} \quad \text{for} \quad \sigma_n \geq 0 \tag{6b}$$

We have assumed that the material flaws causing fracture upon tensile loading are adequately and consistently modeled by Griffith cracks throughout the material volume. The Griffith crack, however, is not the best model for either surface or volume flaws because it is a two-dimensional, through-the-thickness crack. It is for this reason that only Eq. (6a) has been previously used in connection with this crack geometry. More appropriate three-dimensional crack models exist (penny-shaped or elliptical for volume flaws and Griffith notch or semi-elliptical for surface flaws), but the equations were derived herein for a Griffith crack for simplicity and historical considerations.

With these definitions in mind, the Batdorf theory will be outlined next. Further details are available in the references (Refs. 1, 23, 26 to 29). The failure probability of a small uniformly stressed volume of material, ΔV, is the product of two independent probabilities, P_1 and P_2. P_1 is

the probability that a crack exists within that volume which has a critical stress between σ_{cr} and $\sigma_{cr} + d\sigma_{cr}$. The mathematical form of P_1 is

$$P_1 = \Delta V \frac{dN(\sigma_{cr})}{d\sigma_{cr}} d\sigma_{cr} \tag{7}$$

where $N(\sigma_{cr})$ is the Batdrof crack density function which is a material property usually determined from simple uniaxial tests. It is usually assumed that it can be expressed in the form of a Weibull distribution which under certain conditions can be written as a power function

$$N(\sigma_{cr}) = k_B \sigma_{cr}^m \tag{8}$$

where k_B is the Batdorf crack density coefficient.

Fracture depends not only on the existence of the crack with a certain critical strength, but also on the crack orientation with respect to the applied stresses, the magnitude of those stresses, the shape of the crack, and a fracture criterion. In other words, fracture depends on the effective stress and is possible only when the effective stress is greater than the critical stress. Since the effective stress depends on the crack orientation and since the cracks are assumed to be randomly distributed, a second probability function, P_2, is introduced. Specifically, P_2 is the ratio of the angular range of crack orientations for which $\sigma_e \geq \sigma_{cr}$ to the total range of possible crack orientations. Having defined both P_1 and P_2, the failure probability for the entire component can now be written as

$$P_f = 1 - \exp\left[-\int_V dV \int_0^{\sigma_1} \frac{dN}{d\sigma_{cr}} P_2 \, d\sigma_{cr}\right] \tag{9}$$

A computer program (Structural Ceramics Analysis and Reliability Evaluation or SCARE) is under development at NASA Lewis to predict the fast fracture reliability of monolithic ceramics. The program is based on the Batdorf theory as previously described. The program also includes the PIA theory because of its historical significance and for the sake of comparison. Details of the program are contained in the references (Refs. 26 to 28).

NDE'S ROLE IN RELIABILITY TESTING

Background

Stringent testing is needed to verify the Batdorf theory (Ref. 30). An experimental verification program which involves materials characterization, analysis, fracture testing, and NDE has been initiated at NASA Lewis. The program will be briefly described with an emphasis on the importance of NDE to assure valid results. In this application, NDE is utilized to remove physical uncertainty about the material being tested.

An important factor in evaluating the Batdorf theory is the uniformity of the test material. The density of ceramic disks or modulus of rupture

(MOR) bars is usually lower at the core of the specimen than at the periphery (Ref. 31). Nonuniform density causes a nonuniform fracture toughness, thermal expansion coefficient and conductivity, elastic modulus (Refs. 32 and 33), and Poisson's ratio (Ref. 33). This will affect the accuracy of the elastostatic analysis upon which reliability calculations are based. Furthermore, the crack density function $N(\sigma_{cr})$ will vary. In a rigorous testing program these variations can be minimized by characterization of test specimens by NDE as described below.

Approach

One hundred commercially available sintered alpha silicon carbide disks have been examined with conventional and microfocus radiography. The thickness of these disks varied between 1.27 mm (0.050 in.) and 3.81 mm (0.150 in.) and their outside diameter was 50.80 mm (2.00 in.). From these disks, the 45 with the most uniform radiographs (indicative of near uniform density and microstructure) were selected for further surface preparation and subsequent testing. Elaborate machining procedures were performed on these disks to minimize grinding damage and to obtain an isotropic surface finish. In addition they were carefully lapped and polished to an extremely fine mirror surface finish.

After surface preparation is complete, the disks will be subjected to further scrutiny. Proposed methods include acoustic microscopy, ultrasonic velocity and attenuation mappings, and computerized tomography. Acoustic microscopy can provide information about gross manufacturing flaws. Furthermore, ultrasonic velocity imaging is more sensitive to microstructural irregularities such as density variations than radiography (Ref. 34). Because of the long time required for imaging, it has also been decided that computerized tomography be selectively performed to provide more timely results. In any event, it is essential that the MOR bar characterization of the material flaw population accurately represents the material imperfections, and that failure in the disks as well as the MOR specimens be caused by statistically identical phenomena.

After the careful surface preparation and extensive NDE characterization, several disks will be cut into four point loaded MOR bars for uniaxial fracture tests. Selection of the disks for MOR bar testing and the pattern for cutting the bars will depend on the observed density variations and the desire to check isotropic surface behavior. From the MOR bar fracture strengths the Weibull parameters and the Batdorf crack density coefficient can be calculated. It should be noted that when MOR bar tests are employed to calculate these parameters for design purposes, these tests should be conducted with great care to avoid potential experimental errors as discussed by Baratta (Ref. 30). The testing fixtures for this program were designed based on the proposed military standard (Ref. 35). The remaining disks will be fractured under pressure loads which produce a biaxial stress state. Then the failure probability under the biaxial stress state can be calculated and compared to the experimental observations. Fractography is necessary for all the tests to separate the critical flaws into volume and surface flaw populations. Similar room temperature tests will be performed later on silicon nitride specimens.

NDE'S ROLE IN DETERMINING THE STATISTICAL PARAMETERS

Two-Parameter Distribution

A research area which has not yet been discussed in the literature is the possible correlation of the Weibull parameters and the Batdorf crack density coefficient with NDE measurements. Presumably, this could be accomplished by characterizing only one test specimen, rather than the thirty or more required for typical destructive testing. This is true because NDE can look at the variation within one representative specimen by taking measurements at various locations. In contrast, mechanical testing is limited to finding the single worst flaw under appropriate test conditions. However, the crack density functions for both surface and volume flaws must be determined, that is, an NDE method which is sensitive to imperfections only within the volume must be found as well as one which measures surface flaw density. If this could be accomplished, the cost of specimens and of their surface preparation would be greatly reduced. However, it should be noted that the crack density function is not simply the number of cracks per unit volume (or surface area), but it involves the number of cracks with a critical strength less than or equal to given critical stress.

Significance of the Third Parameter

The two parameter Weibull distribution is very popular for characterizing the strength variation of brittle materials because it yields a straight line on a ln ln - ln plot of inverse survival probability versus strength. However, this distribution implies that there is always a nonzero failure probability no matter how small the applied stress. When very high reliability is desired ($P_f << 0.01$), this conservative assumption may lead to inefficient and unaffordable designs.

Shih (Ref. 16) has shown that if data from 20 to 30 test specimens can be represented by a two parameter Weibull distribution then it can also be reasonably represented by several three parameter Weibull distributions. If the desired reliability is within the data range ($0.05 < P_f < 0.95$), any one of these curves will give the same result. However, at low failure probabilities, the allowable stress for each distribution is quite different. The selection of one function rather than another is an arbitrary extrapolation of the available data. The customary choice of $\sigma_u = 0$ will generally lead to conservative designs at low failure probabilities.

Shih's analysis is concerned only with failure probabilities for a single stress state, usually the uniaxial case. When using the third parameter, the theories for multiaxial reliability predictions become more complicated, even for a simple uniaxial stress state. Vardar and Finnie (Ref. 12) have shown that if uniaxial data is used to determine a three-parameter Weibull distribution and then a uniaxial stress state is analyzed by using the Weibull normal stress averaging technique with $\sigma_2 = \sigma_3 = 0$, the result does not agree with original data. This discrepancy occurs because the original Weibull function does not assume any directional dependency whereas the normal stress averaging method presumes a directional variation in the flaw population.

Evans (Ref. 36) has suggested that the three parameter function should be valid only in an equitriaxial tensile stress state (for volume flaws). This is a reasonable assumption since only equitriaxial stress will cause a flaw to be subject to the same stress state (no shear stress) regardless of its orientation to the principal stress axes. Matsuo (Ref. 37) extended this assumption using Batdorf's theory. He showed that the number of cracks per unit volume (crack density function) should be a function of $(\sigma_{cr} - \sigma_u)$ if the three parameter distribution is valid for equitriaxial tension. The analysis also shows that the strength distribution under uniaxial or equibiaxial loads is no longer described by the three-parameter Weibull distribution. In addition, Matsuo plotted the ratio of uniaxial to equibiaxial failure probabilities (for small probabilities) as a function of σ_u and m. As the threshold stress increases, there is a larger difference between uniaxial and equibiaxial fracture probabilities.

Methods to Determine the Third Parameter

There are several methods for determining the threshold stress. Among these, NDE holds special promise. However, verification of the effect of the third parameter in the low failure probability region has not yet been undertaken because of the hundreds of necessary tests.

Several analytical methods for determining σ_u have been discussed by Gregory and Spruill (Ref. 38). A simple method is to modify the ln-ln versus ln plot of inverse survival probability versus strength so that the threshold strength is included on the abscissa $(\sigma - \sigma_u)$, where σ_u is chosen iteratively. Then the value of σ_u which gives the best straight line fit to the data is picked. However, for small sample sizes, considerable judgment is required to pick the best value of σ_u. A second method is to use a least mean squares approximation. In this method, values for σ_u are iteratively selected until the least mean squares difference between the distribution and the statistical sample is sufficiently reduced. McClintock's method of moments has also been used to calculate σ_u. This involves calculating the skewness of the distribution. It has been shown that for accurate representation both McClintock's method and the least mean squares method require at least 100 samples (Ref. 38).

It has often been suggested that a component be proof-tested to determine its threshold strength level so that performance could be guaranteed. The proof test reflects the varying stress state actually present in the component, rather than a constant equitriaxial state, where the location and orientation of the crack would not matter. Thus, the major difficulty with this method is that the obtained threshold strength is indicative of a lower strength limit only in the stress state encountered in the proof test, just as the Weibull normalizing stress is valid only for the stress state tested. Although the determination of this threshold is ultimately desired, it is the material threshold strength rather than the component threshold stress which is necessary for analysis. In addition, it is difficult to economically reproduce in a proof test the stress state encountered in service, especially when thermal stresses are involved. It could also be expensive, especially if many component failures occur in the tests. The potential of subcritical crack growth must also be remembered and care should be taken to avoid this form of additional time dependent material damage.

An alternate method, without the drawbacks noted above, is the use of NDE. NDE can screen out components with large flaws. Assuming that NDE methods can virtually always detect defects larger than a given size then, by taking the weakest possible flaw type and a locally low fracture toughness, a threshold strength can be calculated for a component using LEFM principles. For this application, NDE must have a very high probability of detection and size determination for relatively large flaws on the surface of and within geometrically complex components, rather than having a lower probability of finding a smaller flaw. When making material characterization test with less complicated geometry MOR bars, smaller flaw sizes may be reliably measured and a correspondingly higher threshold strength would be computed. At least two methods of calculating the material parameters are possible. The first approach is to use the higher threshold strength available from the MOR bar and interpret the other material parameters, σ_o and m, from the corresponding specimen fracture data. It should be noted, however, that the presence of σ_u affects the subsequent calculation of the other two parameters. The second method would assume that the threshold strength is limited by what can be found in the actual component and that the material parameters must be calculated as before but with a different value of σ_u. These NDE applications will assuredly minimize some of the intrinsic statistical uncertainties associated with reliability studies, but will require additional investigations.

CONCLUSIONS

A basic review of statistical reliability analysis has been presented. The importance of NDE in assuring valid experimental results by several methods of characterizing test specimens has been addressed. Furthermore, the role of the three-parameter Weibull distribution in the prediction of low failure probabilities has been discussed along with the potential use of NDE methods to obtain both the threshold strengths and the crack density function parameters. Further studies are needed, especially in the area of material properties determination.

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16. Abstract
Basic design concepts for high-performance, monolithic ceramic structural components are addressed. The design of brittle ceramics differs from that of ductile metals because of the inability of ceramic materials to redistribute high local stresses caused by inherent flaws. Random flaw size and orientation requires that a probabilistic analysis be performed in order to determine component reliability. The current trend in probabilistic analysis is to combine linear elastic fracture mechanics concepts with the two parameter Weibull distribution function to predict component reliability under multiaxial stress states. Nondestructive evaluation supports this analytical effort by supplying data during verification testing. It can also help to determine statistical parameters which describe the material strength variation, in particular the material threshold strength (the third Weibull parameter), which in the past has often been taken as zero for simplicity.

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